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EXCELLENCE PROGRAM - SYJC (SCIENCE), 2019-2020

SYNOPSIS

MATHEMATICS & STATISTICS – PART 2

APPLICATION OF DERIVATIVES [6 MARKS FOR H.S.C.]

❖ **Slope of Tangent & Normal to a Curve :**

(i) Slope of Tangent (Gradient) to the curve $y = f(x)$ at $x = a$:

$$\left(\frac{dy}{dx}\right)_{x=a} = f'(a).$$

(ii) Equation of Tangent to the curve $y = f(x)$ at (x_1, y_1) :

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$$

(iii) Normal to the curve $y = f(x)$:

A line through a point P on the curve and perpendicular to the tangent at P is called the Normal to the curve at P.

(iv) Equation of Normal to the curve $y = f(x)$ at (x_1, y_1) :

$$y - y_1 = \left[\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}\right](x - x_1).$$

❖ **Rate Measure & Related Rates :**

The derivative of the variable y w.r.t. x is the rate of change of y w.r.t. x.

Velocity & Acceleration :

If $s = f(t)$ is a displacement of a particle at time t, then

(i) **Velocity :**

$$v = \frac{d^2s}{dt^2} = f''(t).$$

(ii) **Acceleration :**

$$a = \frac{d^2s}{dt} = f'(t)$$

❖ **Approximations :**

$$f(a+h) \approx f(a) + h\,f'(a)$$

❖ **Rolle's Theorem :**

If $y = f(x)$ is a real valued function of a real variable such that

(i) $f(x)$ is continuous on $[a,b]$

(ii) $f(x)$ is differentiable on (a,b)

(iii) $f(a) = f(b)$, then

there exists a real number $c \in (a,b)$ such that $f'(c) = 0$.

❖ **Lagrange's Mean Value Theorem :**

If $y = f(x)$ is a real valued function defined on $[a,b]$ such that

(i) $f(x)$ is continuous on $[a,b]$

(ii) $f(x)$ is differentiable on (a,b), then

there exists at least one point $c \in (a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

❖ **Increasing & Decreasing Functions :**

(i) Increasing Function :

If function $y = f(x)$ is said to be an increasing function in the interval (a,b) if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$ in (a,b).

As we move from left to right along the curve $y = f(x)$ in (a,b), then the curve will be rising.

Rule : If $f'(x) > 0$ for all $x \in (a,b)$ then $f(x)$ is an increasing function in the interval (a,b).

(i) Decreasing Function :

If function $y = f(x)$ is said to be a decreasing function in the interval (a,b) if $f(x_2) < f(x_1)$ whenever $x_2 > x_1$ in (a,b).

As we move from left to right along the curve $y = f(x)$ in (a,b), then the curve will be falling.

Rule : If $f'(x) < 0$ for all $x \in (a,b)$ then $f(x)$ is a decreasing function in the interval (a,b).

❖ **Maxima & Minima :**

I. First Derivative Test :

1. If function f has a local maximum at $x = a$, if

(i) $f'(a) = 0$.

(ii) $f'(a-h) > 0$ and $f'(b+h) > 0$,

i.e. x passes through the point $x = a$ from the left to the right, $f'(x)$ changes sign from positive to negative.

2. If function f has a local minimum at $x=b$, if

   (i) $f'(b) = 0$.

   (ii) $f'(b-h) < 0$ and $f'(a+h) < 0$,

   i.e. x passes through the point $x=b$ from the left to the right, $f'(x)$ changes sign from negative to positive.

II. Second Derivative Test :

1. If function f has a local maximum at $x=a$, if

   (i) f is differentiable at $x=a$ and $f'(a) = 0$ and

   (ii) $f''(a) < 0$.

   Here, $f(a)$ is maximum value of f.

2. If function f has a local minimum at $x=b$, if

   (i) f is differentiable at $x=b$ and $f'(b) = 0$ and

   (ii) $f''(b) > 0$.

   Here, $f(b)$ is minimum value of f.